NPS55-83-022

NAVAL POSTGRADUATE SCHOOL

Monterey, California

AN ANALYSIS OF THE SATELLITE KINEMATICS
MODEL IN THE NAVAL WARFARE
GAMING SYSTEM

by

Rex H. Shudde

September 1983

Approved for public release; distribution unlimited

pared for:
mander
al Electronic Systems Command
PME 120
Washington, DC 20362

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral J. J. Ekelund
Superintendent

David A. Schrady
Provost

This work was supported by the Naval Electronic Systems Command, PME 2Ø.

This report was prepared by:

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NPS55-83-022	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) AN ANALYSIS OF THE SATELLITE KINEMATICS MODEL IN THE NAVAL WARFARE GAMING SYSTEM		5. TYPE OF REPORT & PERIOD COVERED Technical 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Rex H. Shudde		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, CA 93943		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 64503N N0003983WREE905
11. CONTROLLING OFFICE NAME AND ADDRESS Commander, Naval Electronic Systems Cmd PME 120 Washington, DC 20362		12. REPORT DATE September 1983 13. NUMBER OF PAGES 12
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Naval Wargaming System
NWGS
Satellite

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The satellite kinematics model in the Naval Wargaming System (NWGS) is described and critiqued. Suggestions are made for simplifying the existing computer code and for increasing the accuracy of the satellite simulation.

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE

S/N 0102-LF-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

AN ANALYSIS OF THE SATELLITE KINEMATICS MODEL IN THE NAVAL WARFARE GAMING SYSTEM

by

Rex H. Shudde

Naval Postgraduate School
Monterey, California

September 1983

CONTENTS

Page

A. Introduction 1

B. The SCM Kinematics 1

C. Critique 2

D. Summary of Recommendations 2

E. Detailed Recommendations 3

References 10

Appendix: The Quadrant Arctangent Function 11

ABSTRACT

The satellite kinematics model in the Naval Wargaming System (NWGS) is described and critiqued. Suggestions are made for simplifying the existing computer code and for increasing the accuracy of the satellite simulation.

A. Introduction

The M1Ø satellite control model (SCM) in the Naval Warfare Gaming System (NWGS) serves two primary purposes. First, the model performs orbital kinematic computations to position the satellite(s) to the current game time. Second, the model selects either the radar satellite detection model or the elint detection model depending upon the type of satellite being processed. The purpose of this report is to critique the orbital kinematics section of the SCM and to make recommendations for improvement.

B. The SCM Kinematics

Satellite parameter inputs to the SCM, pertinent to the kinematics, consist of the satellite orbital period, the time of last position update and the latitude, longitude and course at the time of last position update. The time interval since the last position update, t_int, is the current game time minus the time of last update. The great circle distance traveled (measured in arc length) since the time of last update is t_int/period. Using spherical trigonometry, the position and course are updated by moving the satellite along the great circle determined by its last position and course, and the arc distance traveled in t_int. The ground position is then corrected by subtracting the amount of the Earth's rotation in t_int from the updated longitude.

In essence, the SCM assumes all satellites to be in unperturbed circular Keplerian orbits about the Earth.

C. Critique

The major limitation of the SCM is that all orbits are assumed circular. Even when this assumption is valid, the major flaw in the existing SCM is the neglect of the precession of the orbit caused by the Earth's oblateness. This precession depends upon the height and inclination of the orbit and can be as much as 10 degrees per day which, in turn, can cause a ground position error up to 600 n. mi. per day. Neither orbital height nor inclination are input parameters to the SCM.

An additional error, caused by the neglect of the Earth's oblateness, is in the computed latitude of the satellite's ground track. At current game time the computed (geocentric) latitude can be in error by as much as 11.5 n. mi. from the true (geographic) latitude at latitudes near 45 degrees.

D. Summary of Recommendations

The sophistication of the SCM model can be increased in several steps. An overview is given below. Details are given in Section E.

1. The programming of the existing SCM can be simplified by using alternate formulas from spherical trigonometry.
2. By including the satellite semimajor axis (which can be derived from its orbital period) and the orbital inclination, the existing SCM can be easily modified to include the orbital precession caused by the Earth's oblateness. Without this inclusion, a ground position error of 600 n.mi. per day can occur.

3. To include noncircular orbits it will be necessary to introduce all of the satellite orbital parameters. The computations involve the solution of Kepler's equation, the transformation of the in-plane coordinates to an inertial system, and the reduction of the coordinates to geocentric latitude and longitude.
4. The satellite's geocentric latitude can be converted to geographic latitude to account for the Earth's oblateness. This conversion can eliminate an 11.5 n.mi. ground position error in the N-S direction. A by-product is the determination of the satellite's height.

The computation of the satellite position using Recommendations 3 and 4 are time consuming and could degrade real time operation of the existing NWGS. It would, however, be possible to precompute and store the satellite positions in a table prior to running the NWGS.

E. Detailed Recommendations

1. Add a two argument quadrant arctangent function, called here 'qatanbf', to the atrigf include file. Details of the qatan function are in the appendix. Then, the M1Ø_satellite_control_ee module can be modified as follows:

   a. Replace lines 2Ø1-2Ø2 by

```
d_long = qatanbf (sinb_dist * sinb_course , cosb_dist *
            cosb_lat - sinb_dist * cosb_course * sinb_lat).
```

b. Replace lines 206-220 by

```
n_course = qatanbf ( sinb_course * cosb_lat ,
              cosb_course * cosb_dist - sinb_dist * sinb_lat).
```

c. With these two modifications, lines 224-231 are no longer required.

These equations take advantage of the general spherical triangle [Ref. 1]. Both d_long and n_course, as computed above, lie in the range of -1 to +1 bams (-pi to +pi). The 4-part formula [Ref. 2] is used to compute n_course directly. The possibility of polar crossing is automatically included.

2. The formula for the precession of the ascending node of the orbit, owing to the Earth's oblateness, is

$$N = N_0 - \frac{(3\ J_2 \cos i)n(T - T_0)}{2a^2},$$

where N is the longitude of the ascending node at time T, N_0 is the longitude of the ascending node at time T_0, J_2 = 0.001082616 is the Earth's second harmonic shape parameter, i is the orbital inclination, n is the satellite motion and a is the semimajor axis. The motion $n = 2\ /P$, where P is the orbital period. Further, a and P are related by

$$P^2 = 4\pi^2 a^3/k^2,$$

where k = 0.0743669161(e.r.)$^{3/2}$/min. is the Earth's gravitational constant. Using these relations, the formula for the node may be written in one of several ways:

$$N = N_0 - 9.9639(a_e/a)^{7/2}(T - T_0) \cos i,$$

where a_e is the Earth's equatorial radius, N is degrees and T is measured in days, or

$$N = N_0 - 216.74\, P^{-7/3}(T - T_0) \cos i,$$

where N is in degrees, and P and T are measured in minutes, or,

$$N = N_0 - 282.83\, P^{-7/3} t\_int \cos i,$$

where N is in bams, and P and t_int are in seconds as is the custom in the NWGS.

Using the last equation, line 235 of M10_satellite_control_ee can be written as

sat_long = sat_long + d_long - (bams_sec + prec) * t_int,

where prec = 282.83 * $P^{-7/3}$ * cos i. Note that prec is a constant for each satellite.

3. Since noncircular orbits do not give rise to uniform motion of the ground track over a non-rotating spherical Earth, the method used in M10_satellite_control_ee cannot be used. In fact, eccentric orbits can give rise to ground tracks containing cusps.

Whatever procedure is used for the orbital computation, it will be necessary to obtain all of the satellite parameters. These are: the semimajor axis a, the eccentricity e, the longitude of the ascending node N_0, the argument of perigee

w_0, the angle of inclination i, and the mean anomaly M_0 for some epoch T_0, or their equivalent. Because of the complexity of the computations, it might be advisable to obtain the orbital elements distributed by NORAD and to use simplest of their existing programs, the SGP model [Ref. 3]. The NORAD elements differ slightly from those given above but they also compensate for orbital decay due to atmospheric drag. Another alternative would be to use equations such as those given in Escobal [Ref. 4]. An outline of the necessary computations is given below.

Set up the global constants:

$k = 0.0743669161$ = gravitational constant,

$J_2 = 0.001082616$ = second harmonic of Earth,

$a_e = 1$ e.r. = Earth's equatorial radius,

$T'_g = 0.0043752695$ rad/min = Earth's rotation rate.

Input a, e, i, w_0, N_0, M_0, and T_0.

Compute the satellite constants:

$$p = a(1 - e^2),$$

$$n = ka^{-3/2}[1 + 1.5\ J_2(1 - e^2)^{-1/2}(1 - 1.5 \sin^2 i)/p^2],$$

$$w' = 1.5\ J_2\ n(2 - 2.5 \sin^2 i)/p^2,$$

$$N' = -\ (1.5\ J_2\ n \cos i)/p^2.$$

The time interval, measured in minutes from T_0, is $T - T_0$, where T is the current time. Compute:

$$w = w_{\emptyset} + w'(T - T_{\emptyset}),$$

$$N = N + N'(T - T_{\emptyset}),$$

$$M = \mathrm{mod}[n(T - T_{\emptyset}),\ 2\pi].$$

Solve Kepler's equation for E:

$$E - e \sin E = M.$$

Compute the in-plane position components, X_w and Y_w, and the radius vector R:

$$X_w = a(\cos E - e),$$

$$Y_w = a(1 - e^2)^{1/2} \sin E,$$

$$R = a(1 - e \cos E).$$

Reduce the in-plane coordinates to equatorial coordinates:

$$P_x = \cos w \cos N - \sin w \sin N \cos i,$$
$$P_y = \cos w \sin N + \sin w \cos N \cos i,$$
$$P_z = \sin w \sin i,$$
$$Q_x = - \sin w \cos N - \cos w \sin N \cos i,$$
$$Q_y = - \sin w \sin N + \cos w \cos N \cos i,$$
$$Q_z = \cos w \sin i,$$
$$X = P_x X_w + Q_x Y_w,$$
$$Y = P_y X_w + Q_y Y_w,$$
$$Z = P_z X_w + Q_z Y_w.$$

Compute the right ascension, RA, and declination, DEC, of the satellite:

$$RA = \text{qatan}(Y,X),$$

$$DEC = \arccos(Z/R).$$

The geocentric latitude of the satellite $L_t' = DEC$.

Compute the unnormalized longitude:

$$L_n' = RA - T_g - T_g't,$$

where t is the total number of minutes elapsed since the year, month and day of $T_\emptyset$, and T_g is the Greenwich sidereal time at the initial time $T_\emptyset$. T_g, measured in degrees, is given by

$$T_g = 99.69\emptyset9833 + (36\emptyset\emptyset\emptyset.7689 + \emptyset.\emptyset\emptyset\emptyset387\emptyset8T_u)T_u,$$

where T_u is given by

$$T_u = (JD - 2415\emptyset2\emptyset.\emptyset)/36525,$$

and JD is the Julian Day Number,

$$JD = 367Y - \text{int}\{7[Y+\text{int}((\mathbf{M} + 9)/12)]/4\}$$
$$+ \text{int}(275\mathbf{M}/9) + D + 1721\emptyset13.5 + UT/24,$$

where Y is the year ($19\emptyset1 \leq Y \leq 2\emptyset99$), **M** is the month ($1 \leq \mathbf{M} \leq 12$), D is the day ($1 \leq D \leq 31$), and UT is the universal time in hours. Then compute the east longitude L, where

$$L_n = \text{mod}(L_n', 2\pi).$$

4. A conversion of geocentric latitude, L_t', to geographic latitude for an oblate Earth can be found in Ref. 4. A slightly improved version is given below. Let

$$a_e = 6378.135 \text{ n. mi.} = \text{Earth's equatorial radius,}$$

$$f = 1/298.26 = \text{Earth's flattening factor,}$$

$$r = a_e R.$$

Let L' = DEL. Then compute

$$r_c = a_e\{[1 - (2f - f^2)]/[1 - (2f - f^2) \cos^2 L']\}^{1/2},$$

$$L_t = \arctan[\tan L'/(1 - f)^2],$$

$$H/r = [1 - (r_c/r)^2 \sin^2(L_t - L']^{1/2} - (r_c/r) \cos(L_t - L'),$$

$$DL' = \arcsin[(H/r) \sin(L_t - L')].$$

Next, let L' = DEL - DL' and recompute from the equation for r_c until L' no longer varies (convergence is rapid). L_t is the geographic longitude of the satellite and H = r(H/r) is the height of the satellite above the oblate Earth.

References

1. Chauvenet, William, "Treatise on Plane and Spherical Trigonometry", 1850.

2. Smart, W. M., "Spherical Astronomy", Cambridge University Press, 1965.

3. Hoots, Felix R. and Roehrich, Ronald L., "Models for Propagation of NORAD Element Sets", Spacetrack Report No. 3, Project Space Track, Aerospace Defense Command, United States Air Force, December 1980.

4. Escobal, Pedro R., "Methods of Orbit Determination", John Wiley & Sons, 1965.

Appendix

The quadrant arctangent function, designated qatan, is a two argument function of rectangular coordinates x and y which returns an arctangent value in the range of $-\pi$ to $+\pi$ depending upon the quadrant of x and y. In FORTRAN this function is called ATAN2. It can be computed as follows:

$$\text{qatan}(x,y) = \text{atan}\{y/[x + e*t(\overset{y=0}{\cancel{x \neq 0}})]\}$$
$$+ \pi*t(x < 0)*[\text{sign}(y) + t(y \neq 0)],$$

where

e = smallest number representable on the computer,

$t(z) = 1$ when z is true,

$t(z) = 0$ when z is false,

$$\text{sign}(y) = \begin{cases} +1 \text{ if } y > 0, \\ 0 \text{ if } y = 0, \\ -1 \text{ if } y < 0. \end{cases}$$

DISTRIBUTION LIST

	NO. OF COPIES
Defense Technical Information Center Cameron Station Alexandria, VA 22314	2
Library, Code 0142 Naval Postgraduate School Monterey, CA 93940	2
Office of Research Administration Code 012A Naval Postgraduate School Monterey, CA 93940	1
Library, Code 55 Naval Postgraduate School Monterey, CA 93940	1
Commander Naval Electronic Systems Command PME 120 Washington, DC 20362	3
Director Center for Wargaming Naval War College Newport, RI 02840 Attn: CDR Ricard Adams	3
Prof. R.H. Shudde, Code 55Su Naval Postgraduate School Monterey, CA 93940	30